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Steady state model for the thermal regimes of shells of airships and hot air balloons

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Abstract—A steady state model of the temperature regime of airships and hot air balloons shells is developed. The model includes three governing equations: the equation of the temperature field of airships or balloons shell, the integral equation for the radiative fluxes on the internal surface of the shell, and the integral equation for the natural convective heat exchange between the shell and the internal gas. In the model the following radiative fluxes on the shell external surface are considered: the direct and the earth reflected solar radiation, the diffuse solar radiation, the infrared radiation of the earth surface and that of the atmosphere. For the calculations of the infrared external radiation the model of the plane layer of the atmosphere is used. The convective heat transfer on the external surface of the shell is considered for the cases of the forced and the natural convection. To solve the mentioned set of the equations the numerical iterative procedure is developed. The model and the numerical procedure are used for the simulation study of the temperature fields of an airship shell under the forced and the natural convective heat transfer.

1. INTRODUCTION

THE DEVELOPMENT of airships and hot air balloons needs the calculations of the temperature fields of the shell and the internal gas for the purpose of the predesign determination of the thermal operating conditions. The model, allowing the evaluation of the average temperatures of the shell and the internal gas of hot air balloons was developed in ref. [1]. The model of ref. [1] includes the integral radiative and convective heat transfer fluxes and does not obtain the temperature field of the shell.

This paper is devoted to the developing of the steady state heat exchange model of the airships and the hot air balloons which includes surface distributions of the radiative and convective heat transfers factors and allows the calculation of the temperature field of the shell and the average temperature of the internal gas. For this purpose the following radiative fluxes are taken into account: the direct and the earth reflected solar radiation, the diffuse solar radiation, the infrared radiation of the earth surface and that of the atmosphere. For the calculations of the infrared external radiation the model of the plane layer of the atmosphere is used. The convective heat transfer on the external surface of the shell of the airship is considered for the cases of the forced and natural convection. The distribution of the forced convection heat transfer coefficient on the external surface of the airship shell is calculated in the approximation of the boundary layer theory. The distribution of the natural convection heat transfer coefficient on the external surface of the airship shell is calculated by means of the natural convection criterion dependence for the local Nusselt number from the horizontal cylinders. The evaluation of the internal heat transfer coefficient inside the shell is obtained from the natural convection criterion dependence for the Nusselt number in the horizontal cylinders.

This model is of practical interest for airship and hot air balloon shell design if the upper or lower operating limits for the temperature of the shell material take place. Besides, the average temperature of the internal gas determines the lifting force and must be known for the predesign evaluations of the altitude control means of apparatus.

2. FORMULATION OF THE STEADY STATE MODEL

The temperature fields of the airships and of the hot air balloons are determined by the radiative and convective heat exchange factors (Fig. 1). The incoming energy includes the solar and the infrared radiations fluxes. The heat removal is determined by the convective and radiative heat transfer. The shells of the airships and hot air balloons can be considered as thermally thin bodies because the shells' cross-sectional Biot number is small:

$$Bi = \frac{(\bar{h}_1 + \bar{h}_2)\delta}{k} \ll 1 \quad (1)$$

where δ is the shell thickness, $\bar{h}_1$ and $\bar{h}_2$ are the mean values of the convective heat transfer coefficients for the external and internal surfaces of the shell and k is the conductivity of the shell.

This condition allows one to neglect the cross-sectional second-order temperature derivative and to present the surface heat exchange conditions in the form of the distributions of heat sources in the temperature field equation.

NOMENCLATURE

- a distance between the radiation reception and emitting points [m]
- a_λ spectral infrared atmosphere absorptivity [m^{-1}]
- As total solar spectrum shell external surface absorptivity
- Bi Biot number
- C_g specific heat of the internal gas [$J\,kg^{-1}\,K^{-1}$]
- C_{sh} specific heat of the shell [$J\,kg^{-1}\,K^{-1}$]
- d_e earth surface albedo
- D_r reduced diameter of the shell [m]
- $\mathbf{e}$ sun direction vector
- $\mathbf{e}_e$ earth direction vector
- E_{ef} radiosity flux intensity [$Wt\,m^{-2}$]
- $E_{b\lambda}$ spectral intensity of the black body radiation [$Wt\,m^{-3}$]
- $E_{e\lambda}$ reflected infrared monochromatic radiation flux of the earth surface [$Wt\,m^{-3}$]
- E_λ^* incident infrared monochromatic radiation flux from the atmosphere on the earth surface level [$Wt\,m^{-3}$]
- E_0 solar radiation flux intensity [$Wt\,m^{-2}$]
- dF_{e-sh} angle factor between the shell point and the earth surface
- F_{e-nm} angle factor between the element and the earth surface
- $F_{nm-n'm'}$ elements angles factors
- g gravitational constant [$m\,s^{-2}$]
- h altitude coordinate [m]
- h^* total average heat transfer coefficient [$Wt\,m^{-2}$]
- h'_0, h''_0 limits of the altitude integrating in RTE [m]
- h_1 external convective heat transfer coefficient [$Wt\,m^{-2}\,K^{-1}$]
- h_2 internal convective heat transfer coefficient [$Wt\,m^{-2}\,K^{-1}$]
- $\bar{h}_1$ mean value of h_1 [$Wt\,m^{-2}\,K^{-1}$]
- $\bar{h}_2$ mean value of h_2 [$Wt\,m^{-2}\,K^{-1}$]
- H altitude of the drift [m]
- k conductance [$Wt\,m^{-1}\,K^{-1}$]
- k_g conductance of the internal gas [$Wt\,m^{-1}\,K^{-1}$]
- l circumferential length [m]
- L characteristic length of the shell [m]
- L_c conductive length of the shell [m]
- $\mathbf{n}_e$ earth surface normal vector
- $\mathbf{n}_1, \mathbf{n}_2$ external and internal shell normal vectors
- Nu_D average Nusselt number
- Nu_l local Nusselt number
- Pr_{ai} Prandtl number of the air
- Q heat flux intensity on the shell surface [$Wt\,m^{-2}$]
- Q_v internal heat source power [Wt]
- Ra_l local Rayleigh number
- $R(\bar{z})$ radial coordinate of the shell surface [m]
- S area of the heat exchange surface of the shell [m^2]
- ΔS_{nm} area of the shell element surface [m^2]
- Δt_h time of the heat exchange conditions changes [s]
- Δt_g thermal time constant of the internal gas [s]
- Δt_{sh} thermal time constant of the shell [s]
- T temperature of the shell [K]
- T_a ambient temperature [K]
- T_e temperature of the earth surface [K]
- T_g average temperature of the internal gas [K]
- T_{nm} elements temperatures [K]
- T_α^* reduced ambient temperature [K]
- ΔT difference between the average shell temperature and the temperature of the air or the internal gas
- U motion speed of the airship [$m\,s^{-1}$]
- V internal volume of the shell [m^3]
- ω, ω' direction vectors
- ω^* vector opposite to ω
- $x, y, z, \bar{x}, \bar{y}, \bar{z}$ Cartesian coordinates
- $\Delta\bar{z}_n$ axial coordinate difference of the shell [m].

Greek symbols

- α angle between the shell surface external vector and the vertical direction
- β polar coordinate
- β_{ai} air thermal expansion coefficient [K^{-1}]
- δ thickness of the shell [m]
- $\Delta\beta_m$ polar coordinate difference of the shell [m]
- $\Delta\lambda_{ir}$ infrared radiation interval [m]
- $\varepsilon_1, \varepsilon_2$ emissivities of the external and internal surfaces of shell
- ε_λ earth surface spectral emissivity
- θ, γ sun direction angles
- κ proportionality coefficient between the solar direct and diffuse radiation
- λ wavelength [m]
- ν_{ai} kinematic viscosity of the air [$m^2\,s^{-1}$]
- ρ_g internal gas density [$kg\,m^{-3}$]
- ρ_{sh} shell density [$kg\,m^{-3}$]
- σ Stefan's constant
- φ, ψ spherical coordinates
- Ω hemispherical solid angle [sr]
- Ω_e solid angle of earth view from the shell point [sr].

Subscripts

- i radiative fluxes index
- n, m elements number indices.

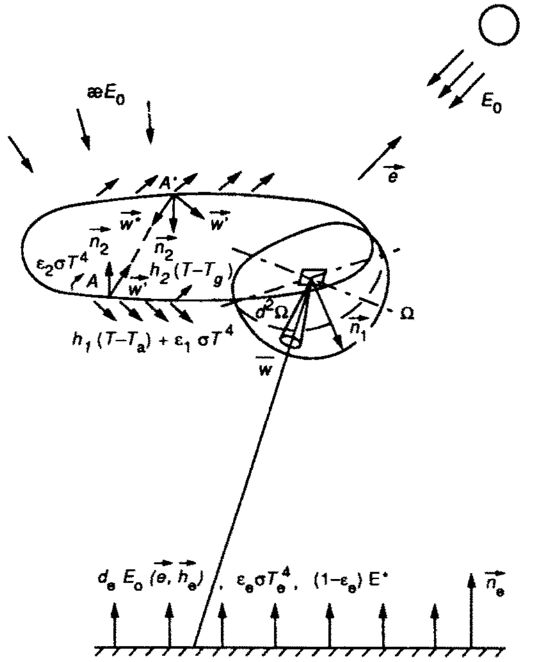

FIG. 1. The scheme for the heat exchange model of the airships and hot air balloons.

The conductive length L_c of the airships or hot air balloons shells is much smaller than the characteristic dimension of the shell L:

$$\frac{L}{L_c} = \frac{L(\bar{h}_1+\bar{h}_2)^{1/2}}{(\delta k)^{1/2}} \gg 1. \qquad (2)$$

This allows one, as shown in ref. [2], to neglect the conductive heat transfer through the cross-section of the shell and the longitudinal second-order derivatives of the temperature in the model.

The characteristic time of the changes of the heat exchange conditions Δt_h assumed to be much larger than the thermal time constants of the shell Δt_{sh}:

$$\Delta t_h \gg \Delta t_{sh} = \frac{\delta \rho_{sh} C_{sh}}{\bar{h}_1+\bar{h}_2} \qquad (3)$$

and that of the internal gas Δt_g:

$$\Delta t_h \gg \Delta t_g = \frac{V\rho_g C_v}{S\bar{h}_2} \qquad (4)$$

where V is the internal gas volume, S is the heat transfer area of the shell surface, ρ_{sh}, ρ_g are the shell and the internal gas densities, respectively, C_{sh}, C_g are the shell and the internal gas specific heats, respectively. These conditions allow one to neglect the time derivatives of the temperature in the heat transfer model.

Taking into account the assumptions (1)–(4) and using Lambert's law for the emissivity, the temperature field model of the shell can be given by the following equation:

$$h_1(T-T_a)+h_2(T-T_g)+\varepsilon_1\sigma T^4 + \varepsilon_2\sigma T^4 = \sum_{i=1}^{I} Q_i + \frac{\varepsilon_2}{\pi}\iint_\Omega d^2\Omega(\omega,\mathbf{n}_2)E_{ef}(\omega) \qquad (5)$$

where ε_1 and ε_2 are the external and the internal emissivities of the shell, respectively, Q_i are the heat fluxes distributions associated with the absorption of the radiation by the external surface of the shell, $\mathbf{n}_2$ is the internal normal vector, ω is the vector of the direction, $E_{ef}(\omega)$ is the radiosity intensity of the internal surface of the shell in the direction of ω and Ω is the hemispherical solid angle.

The radiosity intensity distribution on the internal surface of the shell is described by the integral equation:

$$E_{ef}(\omega) = \varepsilon_2\sigma T^4(\omega) + \frac{1-\varepsilon_2}{\pi}\iint_\Omega d^2\Omega(\omega',\mathbf{n}_2)E_{ef}(\omega') \qquad (6)$$

where $T(\omega)$ is the shell temperature in the point of the shell in the direction of vector ω, ω' is the direction vector over which the integration of radiosity is made for the point of the shell in the direction of ω (Fig. 1).

The average temperature of the internal gas is determined by the integral equation:

$$\iint_S d^2 S h_2(T-T_g) = -Q_v \qquad (7)$$

where Q_v is the power of the internal heat source for the altitude control mean.

The set of the equations (5)–(7) describes the temperature field of the shell and the temperature of the internal gas which depend on the radiative fluxes and the convective heat transfer distributions on the surfaces.

3. RADIATIVE FLUXES MODELS

The distribution of the total heat flux ΣQ_i, included in equation (5), is determined by the radiative fluxes on the external surface of the shell: the direct solar radiation, the earth reflected solar radiation, the diffuse solar radiation, the infrared radiation of the earth surface and that of the atmosphere.

3.1. *Direct solar radiation*

The heat flux associated with the absorption of the direct solar radiation is given by:

$$Q_1 = AsE_0\{(\mathbf{e},\mathbf{n}_1)\}^+ \qquad (8)$$

where E_0 is the direct solar flux which can be evaluated from the semi-empirical correlations, given in ref. [3], or from the solution of the radiation transfer equation, as was reported in ref. [4], $(\mathbf{e}, \mathbf{n}_1)$ is the scalar product of the sun direction vector $\mathbf{e}$ and the external normal vector

$$\mathbf{n}_1, \{v\}^{+} = \begin{cases} v, & v \geqslant 0 \\ 0, & v < 0 \end{cases}$$

is the function which takes into account the self-shadowing of the shell from the direct solar radiation and As is the total solar spectrum absorptivity of the external surface of the shell.

3.2. *Diffuse solar radiation*

The heat flux associated with the absorption of the diffuse solar radiation can be given in the following form:

$$Q_2 = As\kappa E_0 \tag{9}$$

where κ is the proportionality coefficient which is known from semi-empirical correlations, given in ref. [3].

3.3. *Earth reflected solar radiation*

In the approximation which is used in this paper it is assumed that the earth reflected radiation is not diminished considerably by the atmosphere absorption and the scattering along the path from the earth surface to the shell after reflection. Besides, the reflection from the earth surface is assumed to be isotropic.

Taking into account the above mentioned assumptions it is possible to present the absorbed heat flux associated with the earth reflected solar radiation by the following expression:

$$Q_3 = d_e As E_0 (\mathbf{e}, \mathbf{n}_e)\, \mathrm{d}F_{e\text{-sh}} \tag{10}$$

where d_e is the earth albedo, $\mathbf{n}_e$ is the earth surface normal vector, $(\mathbf{e}, \mathbf{n}_e)$ is the scalar product of the sun direction vector and the earth surface normal vector, $\mathrm{d}F_{e\text{-sh}}$ is the angle factor between the shell point and the earth surface which is determined by the following expression:

$$\mathrm{d}F_{e\text{-sh}} = \frac{1}{\pi}\int_{\Omega_e} \mathrm{d}^2\Omega(\omega, \mathbf{n}_1) \tag{11}$$

where Ω_e is the solid angle of the earth surface view from the point of the shell surface.

For the analytical evaluation of the angle factor let us introduce the local coordinate system the origin of which is in the point of the shell surface as is shown in Fig. 2(a). The positive direction of the z-axis coincides with the shell external normal direction. The y-axis resides on the intersection of the shell's tangent plane with the horizontal plane. The x-axis is perpendicular to the plane zOy. The angle φ gives the angle deviation of the direction vector ω from the plane xOy and ψ is the circumferential angle.

Assuming the earth surface to be plane and infinite, let us consider the case when the angle between the z-axis and the vertical, shown in Fig. 2 as α, is within the interval $0 \leqslant \alpha \leqslant \pi/2$. For this case it is evident that the solid angle of the earth surface view conforms to the surface of the unit sphere between the xOy plane and the horizontal plane, as shown in Fig. 2(b). Using the coordinates for the vectors:

$$\omega = \{\cos\varphi\cos\psi, \cos\varphi\sin\psi, \sin\varphi\}, \quad \mathbf{n}_1 = \{0, 0, 1\} \tag{12}$$

the common expression for the differential of the solid angle:

$$\mathrm{d}^2\Omega = \cos\varphi\, \mathrm{d}\varphi\, \mathrm{d}\psi \tag{13}$$

and taking into account the symmetry respectively the plane zOx one can obtain the following expression for the angle factor:

$$\mathrm{d}F_{e\text{-sh}} = \frac{1}{\pi}\int_0^{\pi/2} \mathrm{d}\psi \int_0^{\varphi_{\max}} \mathrm{d}\varphi \cos\varphi \sin\varphi = \frac{1}{\pi}\int_0^{\pi/2} \mathrm{d}\psi \sin^2(\varphi_{\max}(\psi)) \tag{14}$$

where $\varphi_{\max}(\psi)$ is the dependence of the upper limit of the integration over the angle φ on the angle ψ.

The point on the unit semi-sphere surface, conforming to the upper limit of the integration over φ is determined by the intersection of the unit sphere:

$$x^2 + y^2 + z^2 = 1 \tag{15}$$

with the horizontal plane:

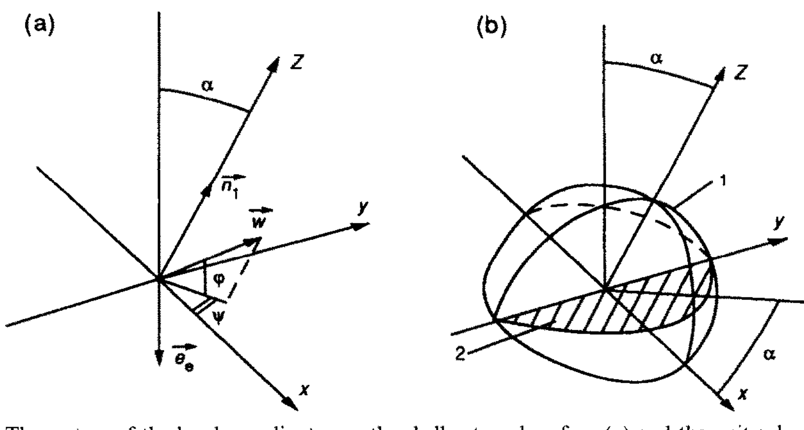

FIG. 2. The system of the local coordinates on the shell external surface (a) and the unit sphere (1) and the horizontal plane scheme (2) for the evaluation of the angle factor between the point of the shell and the earth surface (b).

$$z = x \operatorname{tg} \alpha \tag{16}$$

and with the plane, conforming to the coordinate $\psi = \text{const}$:

$$y = x \operatorname{tg} \psi. \tag{17}$$

The solution of the set of the equations (15)–(17) allows one to obtain the coordinates of this point. Using the z-coordinate of the intersection:

$$z = \operatorname{tg}\alpha(1+\operatorname{tg}^2\alpha+\operatorname{tg}^2\psi)^{-1/2} \tag{18}$$

one can straightforwardly obtain the expression for the upper limit of the integration over the coordinate φ:

$$\varphi_{max}(\psi) = \arcsin z = \arcsin\frac{\operatorname{tg}\alpha}{(1+\operatorname{tg}^2\alpha+\operatorname{tg}^2\psi)^{1/2}}. \tag{19}$$

Substituting (19) in (14) and performing the integration over ψ one obtains:

$$\mathrm{d}F_{e\ sh} = 0.5-0.5(1+\operatorname{tg}^2\alpha)^{-1/2} \quad 0 \leqslant \alpha \leqslant \pi/2. \tag{20}$$

Using the geometrical symmetry and the property for the sum of the angle factors it is possible to show that for the angles within the interval $\pi/2 < \alpha \leqslant \pi$ the angle factor is given as:

$$\mathrm{d}F_{e\ sh} = 0.5+0.5(1+\operatorname{tg}^2\alpha)^{-1/2} \quad \pi/2 \leqslant \alpha \leqslant \pi. \tag{21}$$

Thus, the angle factor for the shell point and the earth surface in the general case can be given by:

$$\mathrm{d}F_{e\ sh} = 0.5 + \begin{cases} -0.5(1+\operatorname{tg}^2\alpha)^{-1/2}, & 0 \leqslant \alpha \leqslant \pi/2 \\ 0.5(1+\operatorname{tg}^2(\pi-\alpha))^{-1/2}, & \pi/2 < \alpha \leqslant \pi. \end{cases} \tag{22}$$

3.4. *Infrared radiation*

Using again the above introduced system of the spherical coordinates it is possible to obtain the following expression for the numerical calculation of the absorbed heat flux, associated with the infrared radiation:

$$Q_4 = \varepsilon_1 \int_0^{2\pi} \mathrm{d}\psi \int_0^{\pi/2} \mathrm{d}\varphi \cos\varphi \sin\varphi \int_{\Delta\lambda_{ir}} \mathrm{d}\lambda I_\lambda(\varphi,\psi) \tag{23}$$

where $\Delta\lambda_{ir}$ is the infrared radiation wavelength interval.

Neglecting the atmosphere scattering and using the model of the plane layer [5], the monochromatic intensity of the incident radiation can be given by the integral form of the radiation transfer equation:

$$I_\lambda(\varphi,\psi) = I_{0\lambda}(\varphi,\psi)\exp\left\{-\int_{h_0'}^{h_0''} a_\lambda(h')\frac{\mathrm{d}h'}{(\boldsymbol{\omega},\mathbf{e}_e)}\right\} + \frac{1}{\pi}\int_{h_0'}^{h_0''} a_\lambda(h')E_{\lambda b}(h')\exp\left\{-\int_{h'}^{h_0''} a_\lambda(h'')\frac{\mathrm{d}h''}{(\boldsymbol{\omega},\mathbf{e}_e)}\right\}\frac{\mathrm{d}h}{(\boldsymbol{\omega},\mathbf{e}_e)} \tag{24}$$

where $E_{\lambda b}(h)$ is the spectral intensity of the black body radiation, $(\boldsymbol{\omega},\mathbf{e}_e)$ is the scalar product of the direction vector $\boldsymbol{\omega}$ and the vector of earth direction $\mathbf{e}_e$ which is given by the expression:

$$(\boldsymbol{\omega},\mathbf{e}_e) = \cos\varphi\cos\psi\sin\alpha - \sin\varphi\cos\alpha, \tag{25}$$

and h is the altitude from the earth surface. The limits of the integration over h are given by:

$$h_0' = \begin{cases} 0, & (\boldsymbol{\omega},\mathbf{e}_e) > 0 \\ \infty, & (\boldsymbol{\omega},\mathbf{e}_e) \leqslant 0 \end{cases}, \quad h_0'' = H \tag{26}$$

where H is the altitude of the airship or hot air balloon drift.

The value of the intensity $I_{0\lambda}(\varphi,\psi)$ on the earth surface, limiting the integration region in the case $(\boldsymbol{\omega},\mathbf{e}_e) > 0$, and on the infinity in the case $(\boldsymbol{\omega},\mathbf{e}_e) \leqslant 0$ can be presented by the expression:

$$I_{0\lambda}(\varphi,\psi) = \begin{cases} \pi^{-1}(E_{e\lambda}(T_e)+(1-\varepsilon_\lambda)E_\lambda^*), & (\boldsymbol{\omega},\mathbf{e}_e) > 0 \\ 0, & (\boldsymbol{\omega},\mathbf{e}_e) \leqslant 0 \end{cases} \tag{27}$$

where $E_{e\lambda} = \varepsilon_\lambda E_{\lambda b}(T_e)$ is the spectral intensity of the earth surface radiation, determined by the earth temperature T_e in accordance with Planck's black body radiation law and by the earth surface spectral emissivity ε_λ, E_λ^* is the monochromatic flux of the downcoming infrared radiation from the above laying atmosphere at the earth surface level [6].

The altitude distributions of the monochromatic atmospheric absorptance and the atmospheric temperature which determine the infrared atmospheric emittance, used in this model, are assumed as known (e.g. from ref. [6]).

4. NUMERICAL APPROXIMATION AND ITERATIVE ALGORITHM

4.1. *Numerical approximation*

For the solution of the problem let us introduce the system of the coordinates whose centre coincides with the centre of the airship (Fig. 3). The $\bar{z}$-axis is directed to the bow of the airship and coincides with the axis of the cylindrical symmetry of the shell. The $\bar{x}$-axis is directed vertically. The $\bar{y}$-axis is directed horizontally, as shown in Fig. 3. The temperature of the shell can be considered in the two-dimensional cylindrical system of the coordinates, presented by the $\bar{z}$-axis and by the angle β, counted from the negative direction of the $\bar{y}$-axis. The sun position is given by the two angles. The angle ϑ gives the deviation of the vector $\mathbf{e}$ from the horizontal plane $\bar{x}O\bar{y}$ and the angle γ presents the deviation of the $\mathbf{e}$-projection on the $\bar{x}O\bar{y}$ plane from the $\bar{y}$-axis. Hence, the sun direction vector $\mathbf{e}$ is given by the coordinates as follows:

$$\mathbf{e} = \{\sin\vartheta, \cos\vartheta\cos\gamma, \cos\vartheta\sin\gamma\}. \tag{28}$$

The shell surface is given by the dependence $R = R(\bar{z})$ which is assumed to be known.

For the approximation of the problem let us introduce the finite elements of the shell ΔS_{nm} ($n = 1,\ldots,N$; $m = 1,\ldots,M$). The boundary coordinates of the elements are given by $\bar{z}_n$ ($n = 0,\ldots,N$)

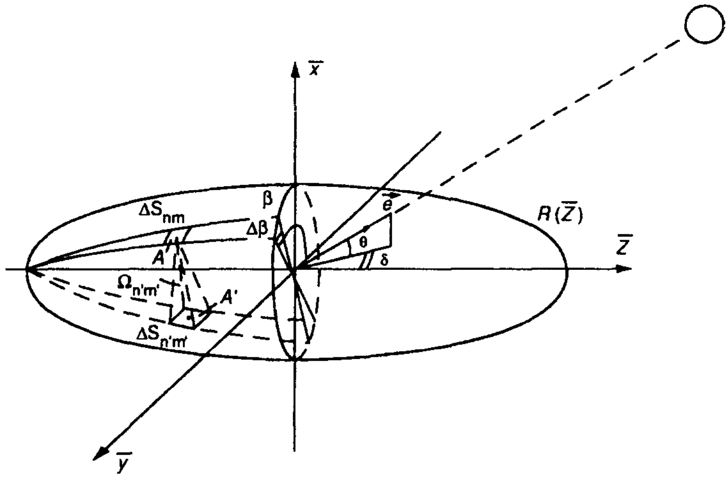

FIG. 3. The scheme for the approximation of the problem.

and β_m $(m = 0, \ldots, M)$. For any element, the temperature T_{nm} and the convective heat exchange coefficients h_{1nm} and h_{2nm} are assumed to be constant.

Performing the integration of the equations (5) and (6) over the element surface ΔS_{nm} and using for the integral (7) the sum over the shell surface, one can obtain the following approximation of the problem:

$$h_{1nm}(T_{nm}-T_{\alpha})+h_{2nm}(T_{nm}-T_{g}) +\varepsilon_1\sigma T_{nm}^4+\varepsilon_2\sigma T_{nm}^4 = \sum_{i=1}^{4}\bar{Q}_{inm} +\varepsilon_2\sum_{n'=1}^{N}\sum_{m'=1}^{M}E_{\mathrm{ef}n'm'}F_{n'm'-nm} \quad (29)$$

$$E_{\mathrm{ef}n'm'} = \varepsilon_2\sigma T_{n'm'}^4+(1-\varepsilon_2)\sum_{n''=1}^{N}\sum_{m''=1}^{M}E_{\mathrm{ef}n''m''}F_{n''m''-n'm'} \quad (30)$$

$$\sum_{n=1}^{N}\sum_{m=1}^{M}h_{2nm}(T_{nm}-T_{g}) = -Q_{v} \quad (31)$$

where the values of the elements areas are expressed through the coordinates β, $\bar{z}$ and the known dependence $R(\bar{z})$ as

$$\Delta S_{nm} = \int_{\beta_m}^{\beta_{m+1}}\int_{\bar{z}_n}^{\bar{z}_{n+1}} R(\bar{z})(1+(\mathrm{d}R/\mathrm{d}\bar{z})^2)^{1/2}\,\mathrm{d}\beta\,\mathrm{d}\bar{z}. \quad (32)$$

The average values of the heat fluxes associated with the radiation on the shell elements are given by the following expressions.

(i) For the direct solar radiation

$$\bar{Q}_{1nm} = \frac{AsE_0}{\Delta S_{nm}}\int_{\beta_m}^{\beta_{m+1}}\int_{\bar{z}_n}^{\bar{z}_{n+1}} R(\bar{z}) \times(1+(\mathrm{d}R/\mathrm{d}\bar{z})^2)^{1/2}\{(\mathbf{e},\mathbf{n}_1)\}^{+}\,\mathrm{d}\beta\,\mathrm{d}\bar{z} \quad (33)$$

where the vector of external normal $\mathbf{n}_1$ is given by

$$\mathbf{n}_1 = \frac{1}{(1+(\mathrm{d}R/\mathrm{d}\bar{z})^2)^{1/2}}\{\cos\beta,\sin\beta,-\mathrm{d}R/\mathrm{d}\bar{z}\}. \quad (34)$$

(ii) For the diffuse solar radiation

$$\bar{Q}_{2nm} = As\kappa E_0. \quad (35)$$

(iii) For the earth reflected solar radiation

$$\bar{Q}_{3nm} = Asd_{\mathrm{e}}E_0(\mathbf{e},\mathbf{n}_{\mathrm{e}})E_{\mathrm{e}\ nm} \quad (36)$$

where the earth-element angle factor is given by

$$F_{\mathrm{e}\ nm} = \frac{1}{\Delta S_{nm}}\int_{\beta_m}^{\beta_{m+1}}\int_{\bar{z}_n}^{\bar{z}_{n+1}}\mathrm{d}F_{\mathrm{e\ sh}}(\alpha) \times R(\bar{z})(1+(\mathrm{d}R/\mathrm{d}\bar{z})^2)^{1/2}\,\mathrm{d}\beta\,\mathrm{d}\bar{z} \quad (37)$$

where the angle α between the external normal vector $\mathbf{n}_1$ and the vertical $\bar{x}$-direction is expressed by:

$$\alpha = \arccos\frac{\cos\beta}{(1+(\mathrm{d}R/\mathrm{d}\bar{z})^2)^{1/2}}. \quad (38)$$

(iv) For the infrared radiation

$$\bar{Q}_{4nm} = \frac{1}{\Delta S_{nm}}\int_{\beta_m}^{\beta_{m+1}}\int_{\bar{z}_n}^{\bar{z}_{n+1}} Q_{4nm}(\beta,\bar{z})R(\bar{z}) \times(1+(\mathrm{d}R/\mathrm{d}\bar{z})^2)^{1/2}\,\mathrm{d}\beta\,\mathrm{d}\bar{z}. \quad (39)$$

4.2. *Elements angle factors*

The elements angle factors, introduced in equations (29) and (30), are determined by the following integrals:

$$F_{n'm'-nm} = \frac{1}{\Delta S_{nm}}\iint_{\Delta S_{nm}}\mathrm{d}^2S\iint_{\Omega_{n'm'}}\pi^{-1}(\omega,\mathbf{n}_2)\,\mathrm{d}^2\Omega. \quad (40)$$

Let us obtain the angle factors values using the above introduced system of the coordinates β and $\bar{z}$.

The differential of solid angle $d^2\Omega$ is expressed through the differential of the surface area d^2S' as:

$$d^2\Omega = \frac{(\omega^*, \mathbf{n}'_2)\, dS'}{a^2} \quad (41)$$

where $\mathbf{n}'_2$ is the vector of the internal normal in the point of intersection of the ω direction with the elements $\Delta S_{n'm'}$, ω^* is the vector opposite to ω, and a is the distance between the reception point A on the element ΔS_{nm} and the emitting point A′ on the element $\Delta S_{n'm'}$ (Fig. 3).

Expressing all values in (40) and (41) through the coordinates β and $\bar{z}$ one can obtain the following expression:

$$F_{n'm'-nm} = \frac{1}{\Delta S_{nm}} \int_{\beta_m}^{\beta_{m+1}} \int_{\bar{z}_n}^{\bar{z}_{n+1}} \int_{\beta_{m'}}^{\beta_{m'+1}} \int_{\bar{z}_{m'}}^{\bar{z}_{n'+1}} \frac{(\omega, \mathbf{n}'_2)(\omega^*, \mathbf{n}_2)}{\pi a^2} \times R(\bar{z})(1+(dR/d\bar{z})^2)^{1/2} R(\bar{z}') \times (1+(dR/d\bar{z}')^2)^{1/2}\, d\beta\, d\bar{z}\, d\beta'\, d\bar{z}' \quad (42)$$

where

$$a^2 = (R(\bar{z})\cos\beta - R(\bar{z}')\cos\beta')^2 + (R(\bar{z})\sin\beta - R(\bar{z}')\sin\beta')^2 + (\bar{z}-\bar{z}')^2. \quad (43)$$

The vectors $\mathbf{n}_1$, $\mathbf{n}_2$, ω have the coordinates, respectively

$$\omega = \frac{1}{a}\{R(\bar{z}')\cos\beta' - R(\bar{z})\cos\beta,\ R(\bar{z})\sin\beta' - R(\bar{z})\sin\beta,\ \bar{z}'-\bar{z}\} \quad (44)$$

$$\mathbf{n}_2 = \frac{1}{(1+(dR/d\bar{z})^2)^{1/2}}\{-\cos\beta, -\sin\beta, dR/d\bar{z}\} \quad (45)$$

$$\mathbf{n}'_2 = \frac{1}{(1+(dR/d\bar{z}')^2)^{1/2}}\{-\cos\beta', -\sin\beta', dR/d\bar{z}'\} \quad (46)$$

where β, $\bar{z}$ are the coordinates of the reception point A and β', $\bar{z}'$ are the coordinates of the emitting point A′.

4.3. *Numerical algorithm*

For the solution of equations (29)–(31) the following numerical iterative algorithm is used.

(1) Introducing the initial data. Computation of the average values of the convective heat transfer coefficients h_{1nm}, h_{2nm}, of the heat fluxes on the elements surface $\bar{Q}_{1nm}$, $\bar{Q}_{2nm}$, $\bar{Q}_{3nm}$, $\bar{Q}_{4nm}$, associated with radiation fluxes.

(2) Computation of the angle factors for the finite elements of the shell $F_{nm-n'm'}$.

(3) Introducing the initial approximation for the value of average temperature of the internal gas T_g.

(4) Introducing the initial approximation for the internal radiosities of the shell elements $E_{efnm-n'm'} = 0$.

(5) The iterative calculation of the elements temperatures T_{nm} by the Newton–Raphson technique [7].

(6) The solution of the set of the linear equations for the elements radiosities using the temperature values obtained in step (5).

(7) The return to step (5) till the convergence of the temperature values.

(8) The calculation of the average temperature of the internal gas from equation (31).

(9) The return to step (4) till the convergence of the average temperature of the internal gas.

5. TEMPERATURE REGIMES OF THE AIRSHIP

The above described model and the iterative procedure were used for the modelling of the temperature fields in the shell of the airship whose surface was approximated by the spheroid. The ratio of the spheroid big axis to the small one for the studied geometry case was chosen to be equal to 4.

5.1. *Convective heat transfer coefficients evaluations*

In Figs. 4(a) and (b) the calculated distributions of the external heat transfer coefficients are shown. Figure 4(a) shows the longitudinal distribution of the heat transfer coefficient under the forced convection for different values of the airship speed. For the calculation the model of the axially symmetric non-compressible airflow was used because the Mach number is assumed to be much smaller than unity [8] for the simulated operational conditions. For the energy transfer equation the isothermal boundary condition was used. The influence of the temperature non-uniformity in the shell on the convective heat transfer distribution was neglected. The distribution of the pressure along the dynamic boundary layer was taken from ref. [8]. The numerical solution of the dynamic and temperature boundary layers was carried out using the code of Patankar and Spalding [9].

As can be seen in Fig. 4(a), the heat transfer coefficient has the maximum in the bow of the airship and is diminishing in the region where the pressure gradient decreases. After the region of the fast decreasing the region of the monotonous decreasing of the convective heat transfer takes place. In the stern of the airship there is another region of the fast decreasing in the distribution of h_1, where the increase of the thermal resistivity of the temperature boundary layer takes place due to the intensive growth of the dynamic boundary layer.

Figure 4(b) shows the circumferential distribution of the heat transfer coefficient under the natural convection for the different values of the temperature difference between the average shell temperature and the airflow temperature. This distribution was calculated using the dependence for the laminar and turbulent natural convection of the horizontal cylinders, developed in ref. [10] by Raithby and Hollands

$$Nu_l = C_t A(\beta) Ra_l^{0.33} \quad (47)$$

where $A(\beta) = 0.71(\cos\ \beta)^{0.33}$ for $\beta = -90°$ to $19°$ and $A(\beta) = (\sin\ \beta)^{0.33}$ for $\beta = 19°–90°$, $C_t = \min(0.14\ Pr_{ai}^{0.087}, 0.15)$, $Pr_{ai} = \mu_{ai}\rho_{ai}c_{ai}/k_{ai}$ is the Prandtl

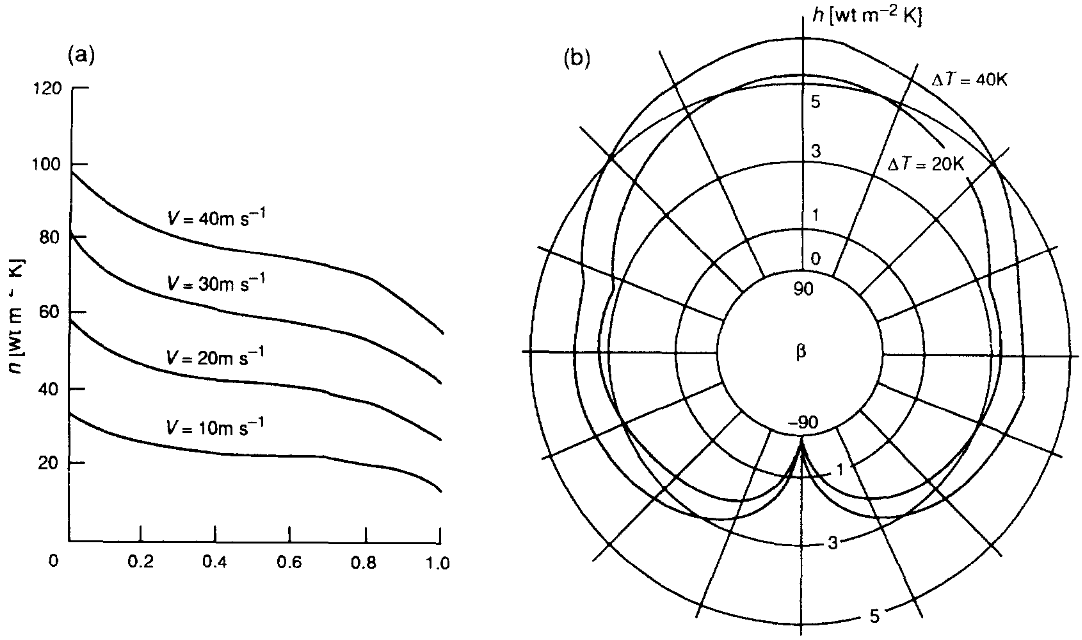

FIG. 4. Convective heat transfer coefficient on the external surface of the shell: (a) the heat transfer coefficient vs dimensionless length from the bow of airship for the case of the forced convection; (b) the heat transfer coefficient vs polar angle β for the case of the natural convection.

number for the air, $Nu_l = h_l l/k_{ai}$ is the dimensionless local Nusselt number and

$$Ra_l = Pr_{ai}\frac{\beta_{ai} g l^3 \Delta T}{\nu_{ai}^2}$$

is the local Rayleigh number, where $l = (\beta + \pi/2)R$ is the circumferential length, counted from the bottom point of the shell, and ΔT is the difference between the air and the average shell temperature.

The internal heat transfer coefficient was estimated using the following dependence for the horizontal cylinders, given in ref. [11]

$$Nu_D = 0.59 Ra_D^{0.25} \qquad (48)$$

where $Nu_D = \bar{h}_2 D_r/k_g$ is the average dimensionless Nusselt number inside the shell and Ra_D is the internal Rayleigh number, which includes the thermophysical properties of the internal gas and the temperature difference between the average shell temperature and the average temperature of the internal gas T_g. For the diameter in Rayleigh number the reduced value D_r was used. This reduced value was obtained from the equivalence of the surfaces and the volumes between the spheroid and the cylinder. The average internal heat transfer coefficient was evaluated to be within the interval $\bar{h}_2 = 3$–5 Wt m^{-2} K^{-1}, depending on the average temperature of the shell.

5.2. *Temperature fields in the shell*

In Figs. 5(a) and (b) the longitudinal temperature distributions in the airship shell under forced convective heat transfer for the airship motion speed $U = 20$ m s^{-1} (Fig. 5(a)) and $U = 1$ m s^{-1} (Fig. 5(b)) for different sun directions are shown. For the case of Figs. 5(a) and (b) the sun direction angles are $\vartheta = 60^\circ$, $\gamma = 90^\circ$. The total solar radiation is $E_0 = 1170$ Wt m^{-2}, the earth reflected solar radiation is $d_e E_0(\mathbf{e}, \mathbf{n}_e) = 420$ Wt m^{-2}, the diffuse solar radiation is $\kappa E_0 = 90$ Wt m^{-2}. The downcoming infrared radiation flux is evaluated to be 355 Wt m^{-2}. The uppercoming infrared radiation flux is evaluated to be 400 Wt m^{-2}. The air temperature is $T_a = 15^\circ$C. The total solar radiation absorptivity of the shell is $As = 0.55$ and the shell emissivities are $\varepsilon_1 = \varepsilon_2 = 0.45$. The internal convective heat transfer coefficient is assumed to be $\bar{h}_2 = 5$ Wt m^{-2} K^{-1}.

The case of Fig. 5(a) shows the temperature field with the maximum value $T_{max} = 299.5$ K. The difference between the maximal and the minimal temperature values is 11.5 K. In the middle part of the shell the longitudinal distribution is nearly uniform, the longitudinal temperature difference does not exceed 3 K. The main temperature differences are localized in the bow and in the stern of the airship. In Fig. 5(b) the longitudinal temperature distributions for $U = 1$ m s^{-1} are shown. The maximal temperature is $T_{max} = 350$ K and the shell maximal temperature difference is 40 K. For these heat exchange conditions the temperature longitudinal uniformity in the middle part of the shell takes place except in the bow and the stern of the airship. The circumferential temperature distribution has rather high non-uniformity due to the non-uniformity of the radiative fluxes on the shell surface.

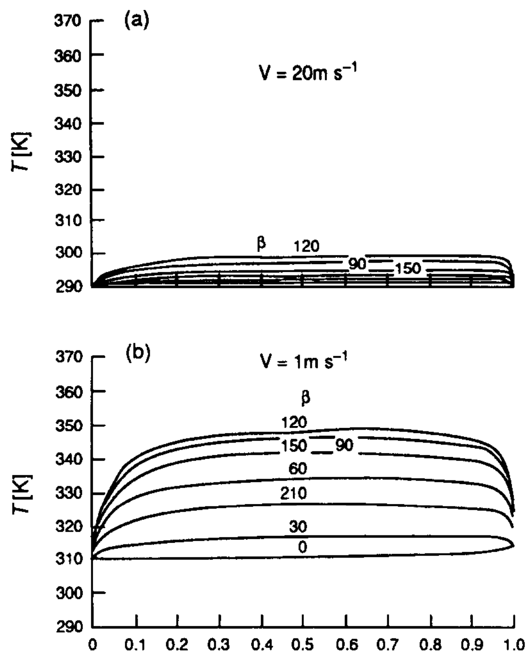

FIG. 5. The temperature distribution for the different polar angles vs dimensionless length from the bow of the airship for the case of the forced convection.

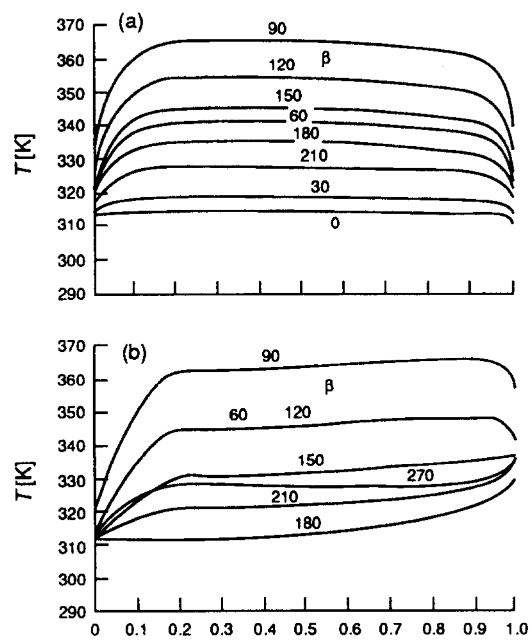

FIG. 6. The temperature distribution for different polar angles vs dimensionless length from the bow of airship for the case of the natural convection.

In Figs. 6(a) and (b) the temperature longitudinal distributions in the shell for the natural convective heat exchange are shown. For Fig. 6(a) the sun direction angles are $\vartheta = 60^\circ$, $\gamma = 90^\circ$. The maximal temperature value is $T_{\max} = 368$ K. For the case of Fig. 6(b) the sun direction angles are $\vartheta = 60^\circ$, $\gamma = 0^\circ$. The maximal temperature is $T_{\max} = 366$ K.

The numerical simulation of the heat exchange of the airship shows that there are two main interplaying effects on the temperature field of the shell and the average temperature of the internal gas. The first one is determined by the ratio of the local heat flux associated with the external radiation to the local convective heat exchange coefficient. If the region of the maximal heat flux coincides with the region of the minimal convective heat transfer the higher temperature field in the shell takes place. This leads to the higher average temperature of the internal gas. For the case of forced convection this situation takes place when the airship is oriented to the sun by the stern.

The second effect is determined by the total heat flux incoming on the surface of the airship which is proportional the shell projection area on the plane, perpendicular to the sun direction. This effect is also associated with the airship orientation relative to the sun direction. The higher temperatures take place when the big axis of the airship is perpendicular to the sun's direction.

5.3. *Contribution of the internal radiative heat transfer*

The contribution of the internal radiative heat transfer in the temperature field of the shell should be pointed out. For high heat transfer on the external surface of the shell under forced convection the temperature field is nearly uniform. In these operating conditions the radiosity field on the internal surface is nearly uniform, too, and can be given by the following approximation:

$$E_{\mathrm{ef}} \approx \sigma \bar{T}^4 = \mathrm{const} \tag{49}$$

where $\bar{T} = S^{-1} \iint_S T \, \mathrm{d}^2 S$ is the average temperature of the shell.

The parametric analysis of the model shows that this approximation of the internal radiative heat transfer is valid only when the following relation is fulfilled:

$$\frac{\bar{h}^* T_{\mathrm{a}}^*}{\max \Sigma Q_i} \geqslant 50 \tag{50}$$

where $\bar{h}^* = \bar{h}_2 + \bar{h}_2$ is the total average convective heat transfer coefficient for both sides of the shell, $T_{\mathrm{a}}^* = (\bar{h}_1 T_{\mathrm{a}} + \bar{h}_2 T_{\mathrm{g}})/(\bar{h}_1 + \bar{h}_2)$ is the reduced ambient temperature near the shell and $\max \Sigma Q_i$ is the maximal heat flux intensity on the shell surface.

For the operating conditions fulfilling the relation (50), the relative error in the temperature field, given by the uniform approximation, does not exceed 1%. The relation (50) is fulfilled when the total heat transfer coefficient is $h^* \geqslant 80$ Wt m^{-2} K^{-1}. This, for the given conditions, takes place if the speed of the airship motion $V \geqslant 100$ m s^{-1}.

If the relation (50) is not fulfilled, the uniform approximation of the radiative heat transfer inside the shell is not valid and gives more considerable errors in the temperature fields of the shell. This takes place for the low motion speeds of the airship and for the natural convection heat exchange regime. All the temperature fields, presented in Figs. 5 and 6 conform to the operating conditions, requiring the calculations of the radiative heat transfer equation inside the shell. The uniform approximation is not valid for the above simulated conditions due to the low convective heat transfer and, as a consequence, to the temperature non-uniformity of the shell.

There is another case of the applicability of the uniform approximation for the internal radiative heat transfer in the shell, which should be indicated here. This case takes place when the geometry of the hot air balloon shell is nearly spherical. For this case the uniform approximation of the internal radiative heat transfer is valid for any operating conditions, because the radiative flux inside the shell is uniform for any temperature fields in the shell due to the spherical geometry, as indicated in ref. [5]. The concrete expression for the internal radiative flux inside the shell for the spherical geometry is presented in ref. [5].

6. SUMMARY AND CONCLUSIONS

In this paper the mathematical model of the steady state thermal regime of airships and hot air balloons shells is developed. The model includes the temperature field equation of the shell, the integral equation for the radiative heat exchange on the internal surface of the shell and the convective heat exchange equation between the shell and the internal gas. The model includes the following radiation fluxes on the external surface of the shell: the direct solar radiation, the earth reflected solar radiation, the diffuse solar radiation, the infrared radiation of the earth surface and that of the atmosphere layer. For the calculation of the convective heat exchange coefficients the known computational technique and criterial dependences are used.

For the solution of the problem the numerical iterative procedure is developed. For the approximation finite elements are used. The numerical algorithm of the solution presents the enclosed iterative procedures of the calculations of the elements temperature values, the elements radiosities and the average temperature of the internal gas.

As an example, the developed model and the numerical procedure are used for the simulation study of the steady state temperature fields of the airship shell under the forced and natural convective heat exchange conditions on. The obtained results show two kinds of the heat exchange interplaying effects on the temperature field of the shell and on the average temperature or the internal gas. The first is determined by the ratio of the local surface heat flux associated with the radiation to the convective heat exchange intensity. The temperature of the shell and of the internal gas would be higher, if the region of maximal heat flux coincides with the region of the minimal convective heat transfer. The second effect is determined by the integral heat flux incoming on the shell surface. The temperature of the shell and of the internal gas would be higher, if the big axis of the airship is perpendicular to the sun direction and the solar energy incoming is maximal.

The contribution of the internal radiative heat transfer, introduced in the model, is studied. It is shown that for the thermal regime with the intensive convective heat exchange and, as a consequence, with the uniform temperature field in the shell the internal radiative heat transfer can be presented by the uniform approximation. In accordance with this approximation the radiosity distribution inside the shell can be given by the constant value, dependent only on the average temperature of the shell. This approximation is valid only for the high motion speeds of the airships and cannot be used for the low speed regime and for the case of the natural convection heat transfer on the external surface of the shell. Hence, the internal radiative heat transfer is shown to be important factor in the heat transfer model of the airships and hot air balloons. The internal radiative heat transfer in general case must be taken into account using the integral equation of the radiative heat transfer, except the case of the very high convective heat transfer from the external surface of the shell and, also, the case, when the shell shape is nearly spherical. In these two special cases the radiative heat transfer can be correctly described by the uniform approximation for the internal radiative flux intensity on the shell surface. This approximation allows one to facilitate considerably the performance of the iterative algorithm and the computations of the temperature fields in the shells.

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LE MODELE STATIONNAIRE POUR LE REGIME THERMIQUE DES ENVELOPPES DES DIRIGEABLES ET DES BALLONS

Résumé—Le modèle stationnaire pour le régime thermique des enveloppes des dirigeables et des ballons est élaboré. Le modèle comprends les equations suivantes: l'equation differentielle de la distribution de la température dans l'enveloppe, l'equation intégrale pour l'échange thermique par la radiation au-dedans de l'enveloppe et l'equation pour l'échange thermique par la convection entre l'enveloppe et le gaz interieur. Le modèle considère les fluxes suivants de la radiation sur la surface exterieure de l'enveloppe: la radiation solaire directe, la radiation solaire reflectée par la surface de la terre, la radiation solaire diffusée, la radiation infrarouge de la terre et de l'atmosphere. Pour l'évaluation de la radiation infrarouge le modèle de la couche plane est usée. L'échange convectif sur la surface exterieure de l'enveloppe est consideré pour les cas de la convection forcée et naturelle. Pour la solution numerique la procédure itérative est élaborée. Le modèle et la procédure itérative sont appliqués pour la simulation des champs de la température dans l'enveloppe du dirigeable en conditions de l'échange thermique par la convection forcée et naturelle.

DAS STATIONÄRE MODELL DER WÄRMEARBEITSWEISE DER HÜLLEN VON LUFTSCHIFFEN UND LUFTKUGELN

Zusammenfassung—Inder Arbeit entwickeltet man des stationäre Modell der Wärmearbeitsweise der Hüllen der Luftschiffen und Luftkugeln. Das Modell einschaltet drei Gleichungen: die Gleichung der temperaturischen Rerteilung in der Luftschiffhülle oder Luftkugelhülle, die Integralgleichung für die Strahlen Ströme an der inner Oberfläche von Hülle und die Integralgleichung des konvektiven Wärmeaustausch zwischen der Hülle und dem inneren Gas. In der Modell betrachtet man die Strahlenströme an der außerlich Oberfläche von der Hülle: die gerade Sonnenstrahlung und reflektierte von der Erdeoberfläche, die diffusione Sonnenstrahlung und Intrarotstrahlung von Erdeoberfläche und Atmosphäre. Man verwendet für die Berechnung von der Infrarotstrauhlung der Atmosphäre das Modell der flachen Schicht. Die konvectiven Wärmeaustausch betrachtet man an der außerlich Oberflöche von der Hülle fur der Falle der erzwungenen und naturlichen konvection. Fur die Lösung des gewiesenen System von Gleichungen wird die Zahlenmäßig iterationene Prozedur entwickelt. Das Modell und die zahlenmäßige Methode nutzt man für die imitationen Forschungen der temperaturischen Feedern in der Hülle des Luftschiffes, in der Bedingungen erzwungener und naturlichen konvektiven Wärmeaustausch aus.

ДИНАМИЧЕСКИЕ МЕХАНИЗМЫ ВОЗНИКНОВЕНИЯ ТЕМПЕРАТУРНЫХ ПЕРЕПАДОВ ПРИ ОДНОРОДНЫХ УСЛОВИЯХ ТЕПЛООБМЕНА

Аннотация—В статье проводится анализ динамических механизмов возникновения, температурных перепадов между конструктивными элементами, находящимися в однородных условиях теплообмена. В стационарных однородных условиях теплообмена температурные перепады между элементами отсутствуют, но возникают в динамических условиях. Причиной возникновения этих перепадов является различие постоянных времени. В работе проведены оценки возникающих температурных перепадов при скачкообразных и периодических изменениях следующих условий теплообмена: температуры потока воздуха, интенсивности тепловыделения, интенсивности конвективной теплоотдачи. Получены аналитические выражения, устанавливающие связь возникающих температурных перепадов с условиями теплообмена. Показано, что температурные перепады малы в двух предельных случаях. Во-первых, при режимах, когда постоянные времени всех элементов много больше периода изменения параметров теплообмена. Во-вторых, при режимах, когда постоянные времени всех элементов много меньше периода изменений условий теплообмена.